Wear of spur and helical gears

Anders Flodin

Stockholm
2000

Doctoral Thesis
Department of Machine Design
Royal Institute of Technology
S-100 44 Stockholm, Sweden
Abstract

Wear of gear flanks is an observed and always present phenomenon in gearboxes. To this point, few investigations addressing the problem of quantifying the amount of wear as well as its distribution have been carried out. In this thesis the amount and distribution of mild wear is predicted using several existing wear models and numerical methods. Both spur- and helical gears are treated. Finally a test series is run on an FZG machine in order to evaluate both the wear development of a gear wheel as well as its distribution. This test is also compared to simulated results for evaluation of the simulation.

Since minor changes of the shape of a surface can lead to significantly increased surface pressures, even mild wear on a gear flank can lead to surface pressures above fatigue limits. In order to simulate and predict the wear of spur gears it is necessary to find the contact forces and the conditions under which the contact take place. An FZG-like test wheel set was therefore modelled using a Winkler elastic foundation model to predict surface behaviour and a modified Archard's equation to determine the wear. Since the wear mechanism on a gear flank can vary, two other types of phenomenological models were tested namely an oxidation model and an adsorption model. The modified Archard's model was found to adequately describe the wear and it was therefore used in simulation of wear of helical gear wheels. The helical gear was modelled with the same parameters as the FZG-like gears but with a helical angle. The tooth itself was regarded as several very thin uncoupled spur gears, which are allowed to deform individually, mounted on a common axis. In this simulation, the surfaces were regarded as Hertz surfaces. It was found that the wear, with time, tended to smooth the differences in wear distribution across root and tip. The transmission error under load was also investigated and found to be reduced by the wear.

An investigation of the real wear development on a set of FZG gear wheels was carried out where plastic replicas of the flanks were made to avoid dismounting the gear wheels. Replicas were made after a time schedule with more stops towards the beginning of the test to monitor the wear in. The replicas as well as the gear teeth were analysed using optical microscopy, SEM and stylus instrument. Focus may be brought upon the very rapid initial wear at the start of the active flank of the pinion. The deviation from the original involute tooth shape found where the number of teeth in mesh changes was also worth noticing. Examining a tooth flank as a whole, a wave pattern was introduced. The results from the analysis were used to evaluate the wear simulations, which was found to well describe the mild wear of a tooth surface. At areas not prevailed by mild wear, such as the root of the pinion, special consideration had to be taken.

Keywords: Contact; Wear; Gears; FZG, Simulation; Wear models, Wear investigation.
Outline of thesis

This thesis concerns modeling, simulation and investigation of wear of spur- and helical gears. It contains an introduction and the following five papers. The papers are referred to by their capital Latin character. The included papers are:


The work carried out in this thesis was initiated and supervised by Professor Sören Andersson. In paper A, B, C, E Anders Flodin performed the main part of the work and Sören Andersson contributed with theories.
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Abstract

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Appendix 1

Appended papers

A. *Simulation of Mild Wear in Spur Gears.*
B. *Wear Simulation of Spur Gears.*
C. *Simulation of Mild Wear in Helical Gears.*
D. *Wear Investigation of Spur Gear Teeth.*
E. *A Simplified Model for Wear Prediction in Helical Gears.*
1 Introduction

This thesis addresses the problem of the wear development of gear teeth surfaces, a matter involving phenomena treated by several branches of the engineering sciences. The thesis deals with both the theoretical side of wear on gear flanks as well as the experimental aspects of the problem.

1.1 Background

Gear manufacturers have so far been occupied with failures due to high root stresses, high surface pressures and hardening cracks; they have neglected investigations of mild wear and its connection to more severe types of damages. Surface fatigue is a known problem and the cure for it has been better and purer materials, smoother surfaces, heat treatments etc. How will mild wear affect the surface pressure and indirectly the fatigue limit? Does the mild wear cause noise and vibration? How is the transmission error under load affected? These questions yet to be fully answered.

If it were possible to calculate the wear distribution already in the design phase, much would be gained since indications as to the performance and service life of the designed product will be obtained. Several researchers have treated the wear of gears, but few have made comparative analyses of wear simulation compared to real tests. Wear is normally treated rather casually using blunt approximations and without reflecting on the effects of wear on the working behaviour of the gears. Wear analysis of tested gears is generally carried out using a scale to indicate the degree of wear. This work differs in that sense due to its relatively sharp focus on the mild wear of gear surfaces and its consequences. The experimental analysis was conducted using top-of-the-line equipment, yielding interesting results as to wear distribution over the tooth surface. Some of the results presented here are not new, since many engineers have worked with gears, though without really focusing on the wear aspect. The fact that gear teeth wear a lot at the root and that the process can be fast is not revolutionary: the question is how much and how fast does it wear and can it be predicted?

Access to cheap computing power has enabled a simulation where the development of the wear over time can be monitored. Also more refined models of wear and surface interaction can be used. Some effects of the mild wear on surface pressures have also been pointed out. Wear investigations of helical gears are rare, and that matter is also dealt with in this thesis. Considerable work has been done on practical aspects, especially by the FZG institute in Munich, Germany but also by universities and private corporations mainly in Europe, the USA and Japan. This work contributes to previous with thorough 3D-stylus profilometry of the surfaces together with powerful visualisation tools as well as a numerical treatment of the wear phenomena on gear flanks using integrated pressures and sliding distances. This has shed some new light on the problem.

1.2 Purpose

The purpose of the first part of this thesis work, paper A and B, was to calculate as accurately as possible the time dependent wear development of gear teeth flanks using known but enhanced modelling methods. Since the wear-in phase is connected to the length of service life, it is of interest to predict whether wear-in will be smooth or harsh. This depends on
running conditions and gear geometry. Another point of interest is the magnitude of surface stresses a gear tooth is subject to after it has been worn in.

High performance gears are normally cut at a helix angle, see figure 1. The advantages are smoother running and a higher load carrying capacity. They are also the type of gears that are of greatest interest to the vehicle industry and thus to society in general.

![Figure 1. Gears cut at a helix angle.](image)

It is therefore interesting to model the wear of helical gears, especially since this is rarely, if at all, found in the literature. This is what paper C and E deals with. Paper C presents an extensive numerical model where the pressure and sliding distance for a small surface element is calculated individually using time integration. Paper E presents a simplified model for simulation of wear of helical gears. Due to the design of helical gears they have milder force transitions during meshing (for an explanation of the term “meshing” see 2.2 Working cycle). This makes it possible to use a simpler less computation intensive model and still obtain satisfactory results thus using less computation time. This simplified model does not predict the wear for spur gears equally well due to the abrupt transitions in indenting force that occurs for this type of gears. Calculations of sliding distances and pressures by time integration are needed to avoid unrealistic results when the number of gear teeth in mesh changes.

Paper D is quite different from the other four. An experimental investigation was carried out on an FZG machine. FZG is an abbreviation for "Forschungsstelle für Zahnräder und Getriebebau" and is situated at TU Munich, Germany. An FZG machine is a back-to-back gear tester for lubrication tests. The goal was to chart the actual wear-in and study the surface changes and damages, and compare the findings from the experimental investigation with simulated results.
1.3 Research approach
The research presented in this thesis is mainly based on theoretical studies and simulations. In paper A, a sophisticated method for calculating the wear of spur gears was developed that yielded more realistic results. Paper B continued this work by incorporating different wear models in order to simulate specific wear mechanisms for spur gears. The question then became what the actual wear was like on the gear tooth surfaces. To understand this, a test had to be run on a suitable machine, the choice being the FZG machine. Since the initial wear was of greatest interest in papers A and B, the wear was more closely monitored at the beginning of the test run. After validation of the test it was concluded that there was a good correlation between the calculated and actual tests results; this suggested it would be meaningful to continue with an examination of the more interesting helical type gear which is more difficult to model. Paper C is the first work in this thesis dealing with helical gears. It found that a numerical approach like that employed in papers A and B made the simulations time consuming to calculate—considerable computer power was needed due to the very large matrices being handled. To obtain sufficient resolution of surface elements to avoid numerical instabilities and other effects, the matrix size became too large for fast calculations. Paper C also concluded that due to a smoother working behavior, a simplification not suitable for wear simulation of spur gears could be feasible for helical gears. That was the hypothesis behind paper E, which was written with focus on quick and accurate wear calculation for helical gears.

2 Gear configuration
Gear wheels are manufactured in a multitude of varieties, some standardised and some not. The most common are probably external cylindrical gears with or without a helix angle, the kind of gears dealt with here.

2.1 Tooth generation
Most gear teeth take their shape from the involute curve, which can be described in parametric form according to figure 2. The idea of using the involute is that it generates a straight contact path when turning the gear wheels which is important since a non-straight contact path, or line of action, see figure 4, generally leads to vibrations at higher rpm. To avoid high contact pressures in the outermost contact regions and to reduce dynamic vibrations, the gears are usually manufactured with tip relief. Tip rounding is employed if there is a risk of the tip edge of the incoming and outgoing tooth being in gear with the working flank of the opposite tooth outside the line of action. Some manufacturer employ root relief as well as tip relief and tip rounding. All these modifications can of course, if not properly designed, lead to a non-straight contact path, which causes noise and vibration. Ideally the contact path should follow the line of action, see figure 4. Crowning across the tooth width is another feature employed to eliminate high contact pressures at the edges of a tooth surface.
\[
\zeta = \frac{d_b}{2} (\cos \beta + \beta \cdot \sin \beta)
\]
\[
\psi = \frac{d_b}{2} (\sin \beta - \beta \cdot \cos \beta)
\]

\[\beta = \text{(Involute angle, see figure to the right)}\]

\[d_b = \text{Base diameter}\]

Figure 2. Generation of tooth profile from the involute curve.

An external cylindrical spur gear wheel—such as the FZG spur gear wheel—and a helical gear wheel are depicted in figure 3. These types are the most commonly used gear wheels. The differences in performance are mainly higher power throughput with the helical gear, due to a higher contact ratio, and less noise if the sum of the lines of contact can be kept constant or near constant. The drawbacks of using helical gear wheels are slightly higher losses and a demand for more advanced manufacturing equipment.

Figure 3. The two different types of gear wheels dealt with in this thesis. Left is a helical gear wheel and to the right is a spur gear wheel.

2.2 Working cycle

The working cycle, referred to in this thesis as the mesh cycle or meshing, is the time that a single tooth spends in contact with the mating tooth during a revolution, see figure 4 below.

The contact situation between two teeth is of the rolling and sliding type. Maximum sliding is achieved at the root of a tooth while ideally there is pure rolling when the contact is on an imaginary line between the wheel centres due to identical velocity vectors for the two contacting surfaces, see figure 4c. The area of contact at this particular moment is referred to as “pitch point” or in a three-dimensional case as “pitch line”.
2.3 Gear problems

The gear wheel is a machine element that has puzzled many engineers, as numerous technological problems arise in a complete mesh cycle. The designer has to make the most durable design that will allow for good economy and performance for the end user. The result of course depends on the segment to which the end user belongs: the different demands pose different problems of which some are discussed below.

Normally gears are put into a case using bearings and shafts. Bearings and shafts deform, bend and twist causing unexpected working behaviour such as jamming, noise and breakdown. The case can add to this since it deforms as well. The case can also amplify the noise if the meshing frequency hits the resonance frequency of the case. Also gear surface texture, generated in the manufacturing process, has been found by Amini [1] to generate unwanted overtones.
Several different forms of stresses are involved. Bending of a tooth causes root stresses, while contact causes surface pressures to vary, especially when the number of teeth pairs in mesh changes. The sliding action, which also varies both in size and direction, causes shear stresses. There exist built in stresses due to manufacturing processes such as hardening and blasting. There are oscillations due to tooth interaction and surface interaction. A new phenomenon, not to be confused with internal stress rupture, is Tooth Interior Fatigue Fracture (TIFF) as described by MackAldener and Olsson [2]. TIFF is a crack propagating from the interior of the tooth resulting in loss of the upper half of the tooth. Despite these drawbacks, gearboxes are still widely used, probably because of their high efficiency and ability to transmit high power, and very compact construction. In many applications gears are maintenance free as well.

Figure 5 shows the relation between some typical types of gear damages and the operating conditions that produce them. As can be seen, the conditions under which the results in this thesis were compiled lie just within the AGMA (American Gear Manufacturers Association) wear limit for a 2000-hour life span.

![Failure limits of a case hardened gear pair.](image)

Standard calculus for root stress and bending fatigue yield conservative results that ensures sufficient tooth strength. The introduction of FEM into gear calculus has made it possible to refine the calculations and optimise the design and in some senses leave the SMS, DIN or AGMA standards. Other mechanisms then limit the service life of the gears. Some of them are not yet fully predictable during design including scuffing, scoring, mild wear, different fatigue mechanisms and damages related to the manufacturing process such as hardening cracks etc.
All of these problems are connected one way or another, but one cannot deal with them all together. Working with gears obviously demands that one seeks out the most influential parameters related to the problem. This work focuses on the gear wheels, or rather the tooth surfaces, ignoring the influences of the housing, shafts and bearings.

3 Simulation of wear in rolling-sliding contacts

Wear in sliding contacts can be a function of several different wear mechanisms such as delamination, abrasion, adhesion or oxidation. Each of them can be described by Archard's wear equation, which is expressed in Equation (1).

\[ Q = \frac{KW}{H} \]  

Where \( Q \) is the worn off volume per unit sliding, \( W \) is the load, \( H \) is the hardness of the material and \( K \) is the wear coefficient which is chosen to distinguish between the wear mechanisms. Many other wear models are derived from Equation (1), which will be shown later.

As long as the relative sliding velocity, or slip, between the surfaces is constant, Equation (1) is quite straightforward. Gears however have a varying degree of slip through their working cycle. This can be dealt with analytically like Andersson [4], who derived sliding distances as a function of contact position on the line of action, or numerically by time integration like Wu and Cheng [5] or Flodin and Andersson [6].

When material is worn off from a surface, deviation from the original shape (i.e. involute for the gears in this thesis) is inevitable. It is therefore necessary to update the surface profile as it wears in order to determine how past wear will affect future wear. This is of great importance when predicting wear development on gear flanks, since the wear is quite different over an individual flank-surface with more wear occurring at the root. Also the tip is considerably worn. At the pitch line there is usually less wear due to lack of sliding action, see figure 6. This lead to a difference in working behaviour for a set of gear as it wears: it is not a steady state process.

![Figure 6. Gear tooth nomenclature.](image)

The most widely used wear model is probably Archard's wear model but several others exist, see the work of Meng [7] who categorised and evaluated a total of 182 wear models. The models used in this work are an oxidation model by Wu and Cheng [8], an adsorption model
founded on the work of Kingsbury [9] and a linear model that can be derived from Archard's wear equation and which has been previously applied to gears by Andersson [4] and Olsson [10] among others. Common to them all is that they require a surface model for determining pressures and a method to calculate sliding distance; this will be dealt with later in the thesis.

In 1987, Lim and Ashby [11] presented their wear mechanism diagrams or wear maps, which show the wear rate, and the regime of dominance, for a number of competing mechanisms of wear. See figure 7. It illustrates how the wear mechanisms depend on normalised pressure and normalised velocity as well as how they relate to each other.

![Figure 7. Wear mechanism diagram. Dry wear for steel on steel contact. a) Wear mechanism diagram after Lim and Ashby [11]. b) 3D-wear map from Põdra [12].](image)

In figure 7b Põdra [12] has constructed a 3D wear map using the wear models presented by Lim and Ashby [11]. From the diagram it can be deduced that the normalised wear (\( \tilde{Q} \)) increases in distinct steps as the wear mechanism alters and that within each mechanism the wear is a linear function of the normalised pressure (\( \tilde{p} \)) and not very dependent on the normalised sliding speed (\( \tilde{v} \)). Only where the sliding speed (\( \tilde{v} \)) is \( 10^2 \) to \( 10^3 \), does the wear (\( \tilde{Q} \)) becomes strongly non-linear. Elsewhere a linear wear model such as Equation (1) should accurately predict the wear well as long as the wear mechanism remains the same; otherwise an adjustment of the wear coefficient is necessary. It should be noted that the diagrams in figures 7a and b are constructed for dry wear in a metal-metal contact. For lubricated contacts, similar regimes have been demonstrated with discrete levels of the wear mechanisms, see the IRG-transition diagram in figure 8 below.
Figure 8. Example of an IRG transition diagram for counterformal contact between steel components fully submerged in an oil bath. From de Gee et al. [13]

The IRG diagram defines the conditions of wear of sliding lubricated concentrated contacts immersed in an oil bath kept at a constant temperature. In regime I the wear coefficient, $k$, is less than $10^{-9}$ (mm$^3$/Nm) and the coefficient of friction normally around 0.02-0.1. The dominating wear mechanism is normally oxidation. In regime II the wear coefficient, $k$, is between $10^{-6}$-$10^{-8}$ (mm$^3$/Nm). Initially the friction coefficient is typically 0.3-0.4 for steel; however, it decreases after running in and can drop down to as low as 0.1. The dominating wear mechanism is oxidation, but adhesive contacts may occur. In regime III the conditions are so severe that adhesive mechanisms of wear are present with unprotected asperity contact. The wear coefficient, $k$, is greater than $10^{-5}$ (mm$^3$/Nm) and the coefficient of friction is high. Gear contacts are subjected to a varying slip and load which means that the wear regime can vary between the regimes in figure 8, as can the wear coefficient.

3.1 Wear models
Since the gear wheels in this study are regarded as slowly rotating, only partial EHL effects can be accounted for and the lubrication mode is mixed or boundary lubrication. This means that asperities will be in contact and bonds, may it be metal-metal bonds or between boundary layers on the surfaces, will be formed and broken in the contact zone. In the model, the contact zone is divided into a number of points –small surface elements– each with a specific area, $A_p$. The real area of contact inside $A_p$ will then be:

$$A_{cp} = \frac{p_p \cdot A_p}{K_r}$$

(2)

where $p_p$ is the mean pressure at a point, and $K_r$ is a constant relating the real contact area to the load applied. Often the hardness ($H$) of the softer surface is made equal to $K_r$ but this implies that the whole contact area is yielding, which may not always be the case. If an asperity has the area $A_{ci}$, the number of asperities in contact in $A_p$ will be:
Assume that the worn off volume of one asperity is proportional to its area and sliding distance, where the sliding distance is the product of time in the contact zone, velocity and probability of contact with another asperity. The worn off volume, $V_i$, will then be:

$$V_i = C_{wm} \cdot A_{ci} \cdot C_{top} \cdot s$$  \hspace{1cm} (4)

where $C_{wm}$ represents the wear mechanism, $C_{top}$ is the probability of contact, $s$ is the sliding distance the point slides against the interacting surface and $A_{ci}$ is the real contact area of asperity $i$. The total worn off volume in $A_P$ will then be the sum of all $V_i$ in $A_P$ according to Equation (5)

$$V_{Ap} = N_n \cdot V_i = \frac{p_p \cdot A_p}{K_r \cdot A_{ci}} \cdot C_{wm} \cdot A_{ci} \cdot C_{top} \cdot s$$  \hspace{1cm} (5)

If $s \rightarrow ds$, (5) will be:

$$\frac{dV_{Ap}}{ds} = \frac{p_p \cdot A_p}{K_r} \cdot C_{wm} \cdot C_{top}$$  \hspace{1cm} (6)

If $C_{top}$ is equal to 1, $K_r$ is equal to the hardness of the softer surface and $C_{wm}$ is a constant, then (5) can be formulated, if summed over every $A_P$, as Archard's wear equation.

$$\frac{V}{s} = K \cdot \frac{W}{H}$$  \hspace{1cm} (7)

where $V$ is the volume of the worn-off material, $W$ is the applied normal load, $H$ is the hardness of the observed surface and $K$ is the dimensionless wear coefficient. For a local small area, i.e. a point, on one of the interacting surfaces, Archard's wear equation can be expressed as:

$$\frac{h}{s} = k \cdot p$$  \hspace{1cm} (8)

where $h$ is the wear depth at the point, $k$ is the wear coefficient, $p$ is the local pressure and $s$ is the sliding distance. Assuming that $h/s$ represents the wear rate at any time i.e., $dh/ds$, and $k$ and $p$ are not dependent on $s$, Equation (8) will be:

$$\frac{dh}{ds} = k \cdot p$$  \hspace{1cm} (9)
which will be named the generalised Archard’s wear equation. The wear model in Equation (9) is a linear model suitable for modelling the delamination wear in figure 7. However, it can be used to model wear mechanisms of almost all types in figure 7, except in the non-linear part of figure 7. The linear wear model has been used in the simulations to describe mild wear of both dry and boundary lubricated sliding surfaces.

Since \( ds = v \cdot dt \), the differential form of Equation (9) becomes:

\[
\frac{dh}{dt} = kpv
\]  

(10)

To emphasise that the equation is solved for each small surface element or point, Equation (10) can be formulated according to Equation 11 where the \( P \) denotes point.

\[
dh_p = k \cdot p_p \cdot v_p \cdot dt
\]  

(11)

When the contact pressures and the sliding speeds are calculated, the wear at each point is determined by a simple Euler integration method, i.e.:

\[
h_{p,n} = h_{p,(n-1)} + \Delta t \cdot k \cdot N \sum_{i=1}^{M} p_{p,i} \cdot v_{p,i}
\]  

(12)

where \( n \) is the current iteration step, \( \Delta t \) is the time step and \( N \) is the number of revolutions per running interval for which the contact conditions can be assumed to be constant. If a point is not in contact during a time step, there will not be any contribution to the wear, since the contact pressure is then equal to zero.

The wear coefficient can, as mentioned, be used to distinguish between the different mechanisms of wear, see figure 7 and 8. Instead of only adapting the wear coefficient to the regime some researchers, for instance Wu and Cheng [8], Kingsbury [9], Quinn [14], Hornbogen [32], have described the mechanism of wear such as spalling of oxide, lack of adsorbed boundary layers or the effects of fracture toughness.

As a result of low sliding speeds, poor EHL effects and low contact temperatures can be expected. This means that junctions of metal or boundary layers can be formed and broken resulting in wear debris and therefore also changes to the gear tooth profile. Oil additives are present to prevent these junctions from forming, but under certain circumstances, such as high temperature and high shear strain, they fail. The mechanisms attaching the lubricant additives to a surface are either chemical or physical adsorption. Wu and Cheng [8] concentrate in their study on physisorption. They have assumed that the wear volume loss is proportional to the product of the real area of asperity contact and the sliding distance and have adapted the results of Kingsbury [9] on adsorbed layers to fit their model. The non-dimensional proportionality constant, known as the wear coefficient, is dependent on the sliding velocity,
the asperity contact temperature, and the properties of the lubricant and contacting solids. This can be represented by Equation (13), which is similar to Equation (7).

\[ V = K \cdot A_c \cdot s \]  

(13)

where \( V \) is the wear volume loss, \( s \) is the sliding distance, \( A_c \) is the real contact area of asperity contacts and \( K \) is the non-dimensional wear coefficient. \( K \) can be broken up into two parameters, one of them specific to the characteristics of the sliding metal pair and the other specific to the characteristics of the boundary lubricant. That is,

\[ K = k_m \cdot \alpha \]  

(14)

where \( k_m \) is the wear coefficient parameter specific to the contacting asperities in relative motion, and \( \alpha \) is the coefficient describing the fraction of the contact that is unprotected by the lubricant. \( \alpha \) is assumed by Kingsbury [9] to be a function of sliding velocity, the asperity contact temperature and the property of the boundary lubricant. The effect of surface roughness makes every parameter a local property. This means that for a local asperity contact, \( i \), the volume loss \( V_i \) by thermal desorption will be

\[ V_i = k_m \cdot \alpha_i (v, T_i) \cdot A_{ci} \cdot s \]  

(15)

Equation (16) is a function varying between 0 and 1, indicating the amount of the contact area that is unprotected by lubricant molecules and thus worn.

\[ \alpha_i (v, T_i) = 1 - \exp \left[ -\frac{X}{v \cdot t_0} \cdot \exp \left( -\frac{E}{R \cdot T_i} \right) \right] \]  

(16)

Where \( t_0 \) is the fundamental time of vibration of a molecule in the adsorbed state, \( X \) is the diameter of a molecule, \( E \) is the heat of adsorption of the lubricant on the surface, \( R \) is the molar gas constant and \( T_i \) is the contact temperature of asperity \( i \). The local wear volume loss considering the thermal desorption becomes:

\[ V_i = k_m \left[ 1 - \exp \left( -\frac{X}{v \cdot t_0} \cdot \exp \left( -\frac{E}{R \cdot T_i} \right) \right) \right] \cdot A_{ci} \cdot s \]  

(17)

which is equivalent to Equation (4) if \( C_{wm} \) is replaced with \( K = k_m \cdot \alpha \) and if \( C_{top} \) is constant and regarded as part of \( k_m \). Equation (17) is now equivalent to the expression derived by Wu and Cheng [8] for adsorptive wear at low temperature. To calculate the total wear rate of a point, a summation of worn asperities over \( A_P \) is necessary as shown in Equation (18).
\[
\frac{dV_{hp}}{ds} = k_r \cdot p_r \cdot A_p \cdot k_m \left[ 1 - \exp \left( \frac{-X}{v \cdot t_0} \cdot \exp \left( -\frac{E}{R \cdot (T_o + 273)} \right) \right) \right] \quad (18)
\]

In Equation (18), \( k_r \) is the inverse of \( K_r \). In many cases it is of greater interest to investigate the wear depth of a point than the worn off volume. The simplest way to do this is to divide Equation (16) by \( A_p \). Equation (18) will then become, with \( k_m \) regarded as part of \( k_r \),

\[
\frac{dh_p}{ds} = k_r \cdot p_r \cdot \left[ 1 - \exp \left( \frac{-X}{v \cdot t_0} \cdot \exp \left( -\frac{E}{R \cdot (T_o + 273)} \right) \right) \right] \quad (19)
\]

At high slide-to-roll ratios, the contact temperature increases and the adsorbed chains of molecules are detached from the contacting surfaces and formation of oxide films may occur. The amount of delamination wear is decreased in favour of oxidation. The linear oxidation model derived below is in fact very nonlinear; its name derives from the reaction mechanism. It is assumed in the simulations that the type of oxide produced is always the same.

In Equation (7), \( W/H \) expresses the real contact area, \( A_c \) and the wear coefficient \( K \) represents the wear mechanism, \( C_{wm} \). Equation (7) can then be rewritten in accordance to Equation (13) as:

\[
V = C_{wm} \cdot A_c \cdot s \quad (20)
\]

where \( C_{wm} \) is

\[
C_{wm} = \frac{A_l \cdot \exp \left( -\frac{Q_l}{R \cdot (T_o + 273)} \right)}{\frac{3}{4} \cdot v \cdot \rho_{Fe} \cdot \frac{M_{O_2}}{M_{Fe}} \cdot \frac{M_{Fe}}{M_{O_2}}} \quad (21)
\]

The expression \( A_l e^{-(Q_l/R \cdot (T_o + 273))} \) in Equation (21) can be regarded as a reaction velocity constant which is mainly dependent on the temperature. If the temperature variation is small \( A_l \) is considered to be constant and proportional to the product of the frequency of collisions and the fraction of molecules in the right position. The factor \( e^{-(Q_l/R \cdot (T_o + 273))} \) is the Boltzmann factor giving the fraction of the molecules that actually achieve the critical energy \( Q_l \).

\( Q_l \) in Equation (21) is constant as long as the reaction mechanism remains the same, which is assumed in the appended paper. The oxide is of the \( \alpha \)-Fe2O3 type that can be expected under lubricated conditions when the temperature is below 400 °C. \( R \) is the molar gas constant, \( A_l \) is the Arrhenius constant for linear oxidation, and \( v \) is the sliding velocity. \( \rho_{Fe} \) is the density of the gear material, \( M_{O_2} \) is the molecular weight of oxygen and \( M_{Fe} \) is the molecular weight of iron. Equation (21) is valid for contact temperatures between approximately 200 °C and 400 °C.

For a local asperity the worn off volume, if linear oxidative wear can be considered to prevail, will be:
\[
V_i = \frac{A_i}{v \cdot \frac{3}{4} \frac{M_{O_2}}{M_{Fe}} \rho_{Fe}} \exp \left[ -\frac{Q_i}{R \cdot (T_o + 273)} \right] \cdot A_{cl} \cdot s
\]  \tag{22}

Following the same procedure as when deriving the adsorption model, the wear rate at a point considering oxidative wear, can be formulated as:

\[
\frac{dV_{Ap}}{ds} = k_r \cdot p_p \cdot A_p \cdot \frac{A_i}{v \cdot \frac{3}{4} \frac{M_{O_2}}{M_{Fe}} \rho_{Fe}} \exp \left[ -\frac{Q_i}{R \cdot (T_o + 273)} \right]
\]  \tag{23}

The wear depth for a point can be formulated as:

\[
\frac{dh_p}{ds} = k_r \cdot p_p \cdot \frac{A_i}{v \cdot \frac{3}{4} \frac{M_{O_2}}{M_{Fe}} \rho_{Fe}} \exp \left[ -\frac{Q_i}{R \cdot (T_o + 273)} \right]
\]  \tag{24}

Since \(ds=v \cdot dt\) and \(k_r\) and \(p_p\) are not dependent on \(t\), Equation (24) can be formulated as:

\[
dh_p = k_r \cdot p_p \cdot \frac{A_i}{v \cdot \frac{3}{4} \frac{M_{O_2}}{M_{Fe}} \rho_{Fe}} \exp \left[ -\frac{Q_i}{R \cdot (T_o + 273)} \right] dt
\]  \tag{25}

Since the shape and number of asperities is unknown, it is difficult to predict the real contact area which ought to be used instead of \(A_p\) in equation (18) as well as in equation (23).

For explanation of symbols, see table 1 below.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h_P)</td>
<td>Wear depth at point P</td>
<td>[m]</td>
</tr>
<tr>
<td>(p_p)</td>
<td>Pressure at point P</td>
<td>[N/m²]</td>
</tr>
<tr>
<td>(k_r)</td>
<td>Wear coefficient</td>
<td>[m³/N]</td>
</tr>
<tr>
<td>(X)</td>
<td>Diameter of adsorbed molecule</td>
<td>[m]</td>
</tr>
<tr>
<td>(v)</td>
<td>Sliding speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>(t_0)</td>
<td>Fundamental time of vibration</td>
<td>[s]</td>
</tr>
<tr>
<td>(E)</td>
<td>Heat of adsorption of lubricant</td>
<td>[J/mol]</td>
</tr>
<tr>
<td>(R)</td>
<td>Molar gas constant</td>
<td>[J/mol K]</td>
</tr>
<tr>
<td>(T_o)</td>
<td>Oxidation temperature</td>
<td>[°C]</td>
</tr>
<tr>
<td>(A_i)</td>
<td>Arrhenius constant</td>
<td>[kg/m²s]</td>
</tr>
<tr>
<td>(Q_i)</td>
<td>Oxidation constant</td>
<td>[J/mol]</td>
</tr>
<tr>
<td>(M_{xx})</td>
<td>Molecular weight of xx</td>
<td></td>
</tr>
<tr>
<td>(\rho_{xx})</td>
<td>Density of xx</td>
<td>[kg/m³]</td>
</tr>
</tbody>
</table>
In a gear contact several different modes of wear are plausible. It is possible to deal with several of them using Archard’s wear Equation (7). In the wear control handbook [15] adhesion-, abrasion-, fatigue- and corrosion-wear are all modelled using Equation (7), but emphasising different aspects in the interpretation of the wear coefficient \(K\). The problem is to know when to use a certain interpretation of \(K\). In figures 7 and 8, different wear regimes are indicated, each with an individual wear coefficient \(k\). One method to determine when to use a particular \(k\) is employed in paper D where transitions between wear mechanisms were found. By introducing the parameter \(C_{\text{severe}}\) into the wear equation to act as a severe wear moderator, an attempt to include the wear transition was made. The function of \(C_{\text{severe}}\) is described by Equation (26).

\[
\frac{dh}{ds} = C_{\text{severe}} \cdot k \cdot p
\]

\[p < p_{\text{lim}} \Rightarrow C_{\text{severe}} = 1\]

\[p > p_{\text{lim}} \Rightarrow C_{\text{severe}} = 100\]  

(26)

Where \(p_{\text{lim}}\) is determined from the expected critical pressure where the wear mechanism changes from mild to severe, see figure 7 and 8. Figure 8 suggests that the difference in \(k\) between wear regimes is a factor of between 10 and 1000. In Equation (26) 100 was chosen, but the value could of course be 1000 or 10 depending on the situation. The model presented in Equation (26) describes the critical wear at the root of the pinion found in paper D, where the surface were cracked and spalled off. Hornbogen [32] have investigated this transition and regarded it as dependent on the relation between the plastic strain produced during asperity interaction and the critical strain at which cracks start to propagate into the material. Hornbogen concluded that empirical tests were necessary to obtain material parameters in order to verify his model.

3.2 Surface models

The gear surfaces can be regarded from several different viewpoints depending on one’s interest. It is possible to include a variety of parameters in a surface model in order to describe plasticity, elasticity, elasto-hydrodynamic effects, impact, waviness etc. In this work the sliding speed is low and the Hertzian pressure below yielding limits and the focus is on the surface pressure. This calls for an elastic model. Waviness is not of interest during the initial wear but is probably be of interest at a later stage, especially since waviness of sliding surfaces is believed to generate noise. For simplicity, impact and surface roughness have been disregarded. The surface roughness could be incorporated into the wear coefficient as an abbott curve a.k.a. bearing length curve in order to simulate the wear-in, this has been done by Põdra [12]. In paper D a strong wear-in effect was noted. The possibility of modelling the wear-in using the abbott curve and Põdra’s approach was, however, given up in favour of Equation (26). Using the abbott curve requires the wear mode to be mild with gradual wear of asperities, but in paper D The wear-in effect is based on a different, more severe mechanism. Only after some time when the wear rate has slowed down, see figure 24, will there be an effect of the gradual wear of asperities.
The elastic foundation model, described by Johnson [16], is used to model the surfaces of the
tooth flanks in papers A, B and D. A contact model according to Hertz is used in papers C and
E. The elastic foundation model can be regarded as a mattress with independent springs. Each
spring is a pressure cell giving the pressure in a point on a flank, see figure 9. The difficulties
of elastic contact stress theory arise because the displacement at any point in the contact
surface depends upon the distribution of pressure throughout the whole contact. This
difficulty is avoided with a simple Winkler elastic foundation model or ‘mattress’ model.

![Figure 9. Principle of a simple Winkler surface model. $F_t$=normal load, $R$=eqv. radius, $K$=elastic modulus of the
foundation, $H$=spring height.](image)

If compared to Hertz' theory, the result obtained from the Winkler bed varies depending on
the relation between $K$ and $H$. The compliance of a point contact is not so well modelled due
to neglect of surface displacement outside the contact. Põdra [12] has investigated this and
concluded that the critical question is the choice for the relation $K/H$ made in order to obtain
compliance between Hertz and Winkler with respect to contact area, maximum contact
pressure or surface compliance. The reasons for using the model is to provide a simple
approximate solution in complex situations where half space theory would be very
cumbersome. Hertz' theory has, as mentioned, been used both as a calculated mean pressure
and as a calculated distributed pressure over the contact zone for a line contact. The
conditions for Hertz' theory are in some sense violated since the surfaces in contact cannot be
considered as half spaces at the beginning and end of a mesh cycle. When calculating wear of
gears there is a dominating factor, namely the sliding distance a surface element is subject to,
the pressure being subordinate. The choice between using the pressures obtained using the
mattress model and Hertz' model is therefore of less importance and calculation speed could
instead be minimised.

FE models have been considered but judged to be too slow for wear calculations for gears.
The results obtained using FEM can be of higher accuracy than with a Winkler model but
when calculating gear wear the sliding distance will predominate, eliminating some of the
advantages using FEM. Another drawback to using FE models for wear calculus is the
computer power required. Lundvall and Klarbring [33] used a powerful SGI computer with a
solution time of 60-75 seconds per time step i.e. 0.002 seconds for a 2D problem. Using the
method described in paper E will solve a 3D problem using significantly less time on a
significantly less powerful laptop PC obtaining similar results.
3.3 Single point observation method
To keep track of the surface elements the single point observation method was used. The single point observation method was introduced by Andersson and Eriksson [17] and has been successfully used by Hugnell [18], Björklund [19] and others. Andersson and Eriksson studied how a point on a gear tooth moved in relation to another point on the opposing tooth, see figure 10. By regarding single points on the teeth, interesting information can be deduced, such as the difference in sliding distance for a local point on the pinion- and gear surfaces, which is discussed below. Local wear and pressure on the flanks can be investigated.

![Figure 10. Principle of single point observation method. S1 and S2 denotes the sliding distances.](image)

The single point observation method naturally relates the wear, pressure and other relevant quantities, directly to the surface thus facilitating the study of local phenomenon on a tooth face. The surface geometry is also easily changed which is important when researching wear. The change of the surface curvature in contact affects the pressures and contact width as well as the load distribution during the meshing. This is important but often neglected and some works treat the wear as a curve fitting procedure [20], making the wear calculation very specific and not general.

3.4 Calculation of sliding distance
At the beginning of a mesh cycle, the two contacting points, P1 and P2, are adjacent, figure 10a. After a small angle increment, \( \alpha \), of the pinion, figure 10b, P2 leaves the contact zone and has then slid the distance \( S_2 \) in relation to P1. After another interval of time, the pinion has rotated an angle \( \beta \) (\( \beta > \alpha \)) and the point P1 on the pinion leaves the contact zone, figure 10c. P1 has then slid the distance \( S_1 \) in relation to P2. Note that \( S_1 \neq S_2 \), which means that the sliding distances must be calculated separately for pinion and gear. Expressions for the calculation of sliding distance that a local point on a tooth surface is subjected to, have been derived by Andersson [4] and are included in this thesis in appendix 1. In the approach used by the author, the sliding distances for a point are computed by numerical integration for each point and calculated separately for the pinion and for gear. This gives a higher accuracy when the number of teeth in mesh are changing, see figure 4. The difference in sliding distance between
3.5 Tooth stiffness

The stiffness of the teeth is of interest when determining the force distribution between the teeth. Wang and Cheng [21] have made FEM simulations of tooth stiffness, and from their result the tooth stiffness can be deduced, see figure 11 below.

The loading position on the tooth surface can be anywhere between tip and root. However, in the case of a tooth pair in contact, the combined stiffness can be considered to be constant without introducing too much error into the final result from the wear calculations. The stiffness has been calculated in papers A, B, and C using the results of Wang and Cheng [21] with the loading position set at around the pitch point. Paper C also uses an empirical model, derived by Simon [22] using FEM calculus. Simon’s model was also used in Paper E. The advantage is that the tooth stiffness varies depending on where the indenting force is applied to the tooth, see figure 12. The effects of varying tooth stiffness on the wear-result is more visible when dealing with helical gears since the contact line will be oblique across the tooth surface. This produces some interesting and very tangible effects on the pressure distribution and wear.

![Figure 11. Dimensionless deflection as a function of gear teeth number and loading position.](image-url)
**3.6 Temperature in gear contacts**

In both the oxidative wear equation and in the adsorption model the temperature is a very significant factor. There have been controversies as to whether it is the general surface temperature or the flash temperature that governs the chemical reactions. In the temperature-dependent wear models discussed earlier, it is the flash temperature that is regarded as governing the formation of oxide and molecules.

In paper B the temperature distribution throughout the contact is calculated in several different ways and then compared. The contact is treated as an infinitely long heat band moving over the surface. The methods compared are the differential heat transfer equation in one dimension, Equation (28), Williams [23] approximation of temperature distribution and a flash temperature equation derived by Tian and Kennedy [24].

Williams Equation (27) is valid for Peclet numbers above 5. See figure 13, which shows the temperature distribution for different Peclet numbers. The Peclet, $Pe$, number can be interpreted as the ratio of surface speed to the thermal diffusion rate, i.e. $Pe = aU/(2\kappa)$ where $a$ is the semi-contact width, $U$ is the absolute sliding velocity and $\kappa$ is the thermal diffusivity. Equation (27) corresponds to the curve for an infinite Peclet number and can be used to approximate analytically the temperature distribution in the contact. Tian and Kennedy have derived their equations from the work of Carslaw and Jaeger [25]. They integrated the steady state temperature expressions using a special variable substitution technique. This avoids singular points obtaining simpler expressions of the temperature distribution for specific geometric and dynamic configurations.
The temperature distribution according to Williams can be approximated to

\[ \theta = \frac{q_o \cdot a}{K_s \cdot \sqrt{\pi}} \cdot \frac{\kappa \cdot (Xpos + a)}{v} \]  

(27)

where \( q_o = \mu \cdot p_{mean} \cdot v \) and \( Xpos \) is the position of the point in the contact. \( p_{mean} \) is the mean pressure in the contact and \( v \) is the velocity. \( K_s \) and \( \kappa \) are the thermal conductivity and thermal diffusivity respectively. Equation (27) leads to a temperature rise equal to the infinite Peclet number in figure 13. A simulation under conditions as described in paper B will use Peclet numbers ranging from around 2 down to 0.1 around the pitch point.

Equation (28) is the heat transfer equation, which only takes account of perpendicular heat transfer into the material. \( T \) is temperature, \( t \) is time and \( x \) is distance into the material.

\[ \frac{\partial T}{\partial t} = \kappa \cdot \left( \frac{\partial^2 T}{\partial x^2} \right) \]  

(28)

Equation (28) is solved using a simple finite difference technique using a few special conditions described in paper B.

Equation (29) is the maximum contact temperature according to Tian and Kennedy previously discussed. Here \( q_o \) is the energy input, \( a \) is the semi contact width, \( Pe \) is the Peclet number and \( \delta_{theory} \) is the division of heat between the bodies in contact and calculated according to Archard and Rowntree [26].
\[ T_{\text{max}} = \frac{2 \cdot a \cdot q_0 \cdot \delta_{\text{theory}}}{K_s \sqrt{\pi (1 + Pe)}} \]  

(29)

All these methods for temperature calculation indicated that in the case of slowly rotating gears, one model is as good as another.

3.7 Modelling wear of gears in 3 dimensions

As long as a spur gear with no helix angle is modelled, effects across the tooth width can be neglected without violating the laws of contact mechanics too much. Boundary effects do exist and the contact is not an infinitely wide line contact even though it is treated as one. For most of the surface of any tooth, the 2D assumption can be considered valid. Helical gears are a different matter. Here the line contact is oblique over the tooth surface and is therefore subject to different degrees of stiffness. To determine the load distribution across the width of the tooth face, a slicing method introduced by Hanes and Ollerton [27] was used in paper C, as well as a model with independent and uncoupled thin teeth mounted on a mutual gear hub used in paper E, see figure 14 below.

![Figure 14. Helical gear wheel demonstrating the principles of oblique contact lines and the slicing method](image)

The sliding between helical teeth is determined by the same equations used for gears without a helix angle, as long as the axes are parallel.

4 Wear testing

There are several different methods for tribology testing, of which the pin-on-disk method probably is the most common for investigating wear. Depending on what is being investigated, different machines are used. When testing oils, a twin disc machine or a four-ball machine may be used –or any of several other alternatives. The FZG machine, developed for investigation of oils, is standardised according to DIN 51 354. By weighing the wheel before and after operation, material wear may be estimated. The origin of the wear debris, however, cannot be determined in this manner. The less the material that has worn off the better the oil. Extensive research to improve the FZG test is being carried out in Germany at the FZG, Faculty of Machine Engineering, TU Munich [28].
4.1 Wear testing of gears

Paper D describes wear testing on an FZG back-to-back gear test rig but differs from the work of Höhn [28], whose work concerns classifying oils with respect to their wear resistance. In paper D, the aim is to determine the wear distribution on a tooth surface with respect to time, and whether it can be predicted. If it can be predicted, the tooth could be designed to minimise the wear thus extending the fatigue limit. Figure 15 below shows a sketch of an FZG rig. According to Höhn [29], the main advantage of the FZG rig is that power is not taken from the system; therefore only a relatively small amount of input power is required to compensate for the losses in the system.

![Sketch of FZG back-to-back gear test rig.](image)

The controlled quantities in the basic FZG test are the rpm, oil temperature and quality including the level of filtering—and load. Pre-tensioning the gears against each other by torsion of the load clutch controls the load, see figure 15. The drive gear counteracts the torsion and the shaft connected to the gear is very thick and therefore stiff, as can be seen in figure 15. To permit wear monitoring, the FZG machine was periodically stopped according to a schedule with shorter intervals towards the beginning, see paper D for details. At each halt the wheels were cleaned and degreased, and a mould of putty and aluminium was built around two gear spaces at three positions on each wheel, see figure 16.
A cold-curing resin with a methylmethacrylate base, supplied as powder and liquid, designed for taking accurate surface impressions, was poured into the mould. After hardening it was removed and examined using optic and stylus instrument. This method and material have been tested by Ohlsson [30] and found satisfactory. Typical measurement uncertainty for an integrating 3D-amplitude parameter, $S_a$, is around 5%.

Another replica method was used by Andersson [4] on gear flanks where acetyl cellulose films are soaked in acetone and then put on the surface. After evaporation of the acetone the film can easily be removed and examined with various instruments. The form of a tooth is not preserved but the different features of the surface such as roughness and cracks can be investigated. Another feature appreciated is the semi-transparency of the films during examination with optical microscope.

4.2 Analysis of wear on tooth flanks
The investigation of the wear was done using a 3D stylus instrument and both SEM and traditional optical microscopy. To visualise the wear from a measurement, the curved form of the original surface must first be removed, revealing waviness and surface roughness. This was done by least square fitting to a third-order polynomial. Only the real teeth were examined with an optical microscope and SEM. With sputtering techniques it would be possible to investigate the plastic mouldings as well. In paper D, examining the mouldings with SEM was judged to be of less significance, since the pieces were examined using a stylus instrument. SEM was regarded as a complement and only used with the real gear teeth.

The resulting measurements showed that a lot of wear took place at the root and top, this was anticipated since both simulation as well as observations from other gear wheels have indicated this. Also found was a waviness of the surface, generated by the wear. Some of the waviness could be directly related to the load change as the number of teeth in mesh changes.
Other irregularities found were more difficult to trace back to their origins, dynamic effects possibly being the reason.

**4.3 A piece of advice from experience**

A word of advice for those thinking of doing mouldings of gear teeth: avoid the Technovit 3040. Technovit 3040 was used for the following reasons: It has been shown to give good results [30]—but other cold curing resins could perform as well—and it works well with stylus instruments.

However, silicone-based resins would be much easier to handle: they are more or less odourless—which Technovit 3040 is not—do not splinter, are easy to remove when spilled, and do not build up high inner tensions during curing that makes it fly off during removal from the gear space. They probably need sputtering when being examined with optic topography instruments since they can not be examined using stylus instrument due to the softness of the silicone material, but it is worth the effort to avoid damaging a gearbox. The FZG gearbox is forgiving and can be cleaned whereas regular gearboxes are rather tight with only small inspection hatches to work with.

**5. Results and conclusions**

The importance of taking wear into considering in the design of gear wheels is obvious, yet rarely respected. Lack of good engineering tools for wear calculations and a widespread belief that mild wear is beneficial are some of the reasons for this. Design standards, such as SMS 1871, very crudely over dimension the teeth to suit them to unpredictable working conditions and does not consider mild wear. However, as in the case of gears, other important factors such as material purity and production equipment are steadily improving. It is therefore increasingly important to take wear into consideration now that tooth optimisation is possible.

Paper A describes an attempt to simulate the wear using sliding distances for discrete surface elements which were calculated by integrating the sliding speed for each time increment. The pressure that a surface element, or point, was subjected to was not the mean pressure of the contact, but dependent on the position of a cell within the contact width. With sufficiently small time increments a point would be subjected to the pressure distribution in the contact.

This lead to some improvements in the wear curves pictured below in figure 17. The wear calculations showed more realistic results regarding the transitions between the number of teeth pairs in mesh. In earlier simulations they where very sharply pointed due to overly drastic simplifications. There was also a very distinct wear-in effect. The wear literally stops at the root and starts to work on the part of the flank closer to the pitch point. Moreover, at the top of the flank there is a significant difference with a drastic increase in wear. This increase was also found to be true in paper D and not modelled very well in the simplified simulation presented in paper A and by Andersson and Eriksson [17].
A hypothesis tested was that even mild wear might well be an initiator for more severe forms of gear failure. Since the wear, whether mild or severe, leads to surface waviness, it can be expected to have a negative impact on the surface pressure, resulting in fatigue limits being exceeded. This can be a rapid process as well. If the teeth are being optimised using more advanced methods, wear must be considered in more detail.

The wear was found to affect the load distribution between the teeth and, as mentioned, the surface pressure, since the bodies in contact locally changes their curvature as well as the amount of load the contact can carry. After sufficient wear, surface pressures above acceptable levels were calculated, see figure 18 below.

Paper B describes an investigation of some different wear models and their suitability for wear calculation of gears. As mentioned earlier, depending on the load, lubrication and sliding speed, different wear mechanisms will prevail, see figures 7 and 8. These mechanisms are oxidation and adhesion and they were compared to the linear model from paper A. The expressions derived to model the wear included contact temperature, which was then calculated using three different approaches.
It was found that under the simulated conditions all the temperature calculation methods yielded very similar results. See figure 19 below. It was also found that the linear model gave good results and was deemed the best model of the three. The oxidation model was found to be unsuitable and the adsorption model was judged acceptable, though it required more calculation time and was therefore slower than the linear model to use, see figure 20.

![Figure 19](image-url)

*Figure 19. Maximum contact temperature rise through mesh using different approaches.*

![Figure 20](image-url)

*Figure 20. Comparison of linear wear model and adsorption model.*

Papers C and E deals with helical gears and their initial wear. Since the working behaviour of helical gears is different [31] compared to spur gears, it was of interest to investigate the mechanisms by which helical gears wear. No serious investigation of this subject was found
in the literature. Olsson [10], for example only touched on the subject briefly. Helical gears are also commercially more interesting: they are generally preferred to spur gear wheels due to their higher loading capacity and generally lower noise levels. It can be concluded that, wear distribution across the tooth width is initially uneven, especially at root and tip. In the simulated gears this uneven wear distribution was very pronounced. By adjusting the helix angle, tooth width, etc., the variation in wear at root and tip can be minimised. After running in, wear distribution becomes more even, see figure 21.

![Figure 21. Wear curves for helical gears. a)First wear cycle, More wear found on slice one which enters the mesh first. b) Wear distribution after $6 \cdot 10^5$ revolutions, the wear distribution at the root changes, becoming more symmetrical at the root.](image)

This asymmetrical pattern can be seen in the works of Vedmar [31] and Olsson [10] who studied the contact load for external and internal helical gears during meshing. The change in contact pressure due to wear can be seen in figure 22 below.

![Figure 22. Mean pressure distribution for a helical gear wheel. a) Unworn gear teeth. b) After $6 \cdot 10^5$ revolutions of the pinion the surface pressure have changed dramatically.](image)

It is interesting to observe the pressure distribution in figure 22 and how it changes with wear. After some operation, there is a pressure maximum on the side-edge at the pitch point. This illustrates well the importance of longitudinal crowning of helical gear wheels. Initially there
is a maximum at the first point of contact but due to the wear it becomes a minimum. This also illustrates the importance of tip and root relief in order to avoid damages, as shown in figure 27.

The wear on the gear teeth leads to both desirable and undesirable effects, with pressure decreasing on some parts of the tooth surface while increasing on other parts. Manufacturing errors are sometimes neutralised, and gear tooth impact can be reduced due to the material removal at the root and top. The effects of wear on the transmission error under load (LTE) is illustrated below in figure 23. As can be seen, the transmission error is helped by the wear since the variation is reduced.

Figure 23. Transmission error before and after wear of helical gear. Bold line shows LTE before wear. To the right is a sketch defining the LTE as the displacement of the contact along the line of action.

On the other hand, a spur gear tooth with an optimum shape that minimised the LTE would after some wear show a higher LTE since the shape of the tooth, due to wear, would differ from the original unworn optimal shape.

The conclusions of paper C was that the wear is more or less asymmetrically distributed across the tooth width. The tooth stiffness is a more influential parameter when modelling wear in helical gear contacts; consequently, a good tooth stiffness model is important. The transitions when a helical gear tooth enters and exits the line of action are smoother than for the spur gear tooth. The simulation method used in paper C was found to be slow due to an iterative method for determining load and pressure distribution. The resolution of points or surface elements required considerable computer power, counter acting the aim of rapid but sufficiently accurate simulations. However, since the transitions between the number of teeth pairs in mesh were found to be mild in simulation of helical gears, the simplifications that were found unfit in paper A could perhaps be used.

Paper E deals with a simplified method for wear simulation for helical gear. The simplifications were taken from paper A, which used mean pressures and Anderssons’s [4] analytical expressions to calculate the sliding distance as a comparison with the improved
simulation model in paper A. The conclusions of paper C were strengthened by the results of paper E. Using the simplified method a lot of new information could be calculated, and quickly as well. The results were in accordance with previous results, but the simulations could be run to greater wear depths without stability problems. The main conclusions of paper E were that some boundary effects were not captured with the simplified model. The simplified model in paper A did not capture the same boundary effects either. The sliding distance was also found to be the most influential parameter, which can be said for all the simulations of wear in gear contacts.

In paper D an experimental study was carried out in order to verify the previous simulations of spur gears. A set of FZG gearwheels were run and stopped at intervals, see chapter 4. The intervals were chosen so that the initial stops were more frequent to facilitate investigating the wear-in. The machine was stopped after 15 minutes; then after 30 minutes, 1 h, 2h, 4h, 8h, 16h, 32h and 64h, the time totalling 128 hours. At each halt plastic casts were made of the teeth’s surfaces and analysed using mainly a 3D stylus instrument. From the measurements it was concluded that the wear in was very fast, see figure 24 below, and that the wear in mechanism was due to severe wear at the root.

![Figure 24. Comparison of simulated and measured root wear.](image)

The two measured curves are from teeth next to one another. This illustrates the difference in wear rate for different teeth on the same gear wheel. The difference in wear rate between the teeth plotted in figure 24 is rather large. Other teeth on the same gear wheel showed wear rates within these two extremes.

It was also found that the wear introduced waviness on the surface, see figure 25 which is a measurement of the dedendum of a pinion tooth.
In figure 25 it is possible to discern the severe wear at the root and the increase in wear where the number of teeth in mesh changes. The pitchline is also indicated, with damages forming dimples across the tooth width. In paper D it was found that the measured wear on a spur gear correlated with that of the simulation. Features found in the simulations can be traced back to the surface. The form and the absolute wear depths are on some parts of the tooth very well modelled. The root is as mentioned exposed to very aggressive delamination wear, see figure 27 below. As a method to model this, Equation (26) was introduced, see figure 26 where the simulated and measured wear is compared.

![Figure 25. Wear induced waviness at the dedendum of a tooth](image)

![Figure 26. Comparison between calculation of wear close to the root and measured wear. The sudden peak in simulated wear at 1 mm on the horizontal axis is due to a hundred-fold increase in the wear coefficient since the pressures are exceeding 2000 MPa, see Equation (26).](image)
When working experimentally it is always good practice to complement an observation method, be it observation by stylus instrument or microscope, with another type of method. In paper D, as a complement to stylus measurements, optical microscope and SEM were used.

![Figure 27](image)

*Figure 27. a) Section through tooth. Wear at start of active flank after 128 hours 100x magnification. Light part in figure is tooth, black is matrix substrate. b) SEM picture of damages at start of active flank. c) SEM picture of individual damage at root with striation marks. d) Crack surfacing, will form a metal flake and spall off. The crack can be seen in the c) photo. 128 hours of running.*

By doing so, elements of uncertainty in the stylus measurements were avoided and new information was gathered. Figure 27 illustrates obvious evidence of the wear at the root is found supporting the stylus measurements; the SEM later revealed the wear mechanism to be a severe form of delamination wear. The main conclusions of paper D were that moderate to severe wear occurs at the root, tip and start of single tooth meshing and moderate to mild wear occurs at the pitch point and end of single tooth meshing. It was also concluded that the simulation predicts this behaviour and that the prediction is very good at the tip and start of single tooth meshing.

Gear failures are often unique and depend on the prevailing conditions, which sometimes can alter even between gears that are supposed to be identical. To investigate gear failures one cannot take the gear out of its context. Still this is what is done in this thesis, drawing conclusions from idealised simulations and tests. But since investigation of every unique case is impossible, generalising is unavoidable. The wear phenomena between gear flanks are thus by no means fully investigated with this work. Some light has been shed on the problem though and the main contributions and conclusions of this work are:
1. It has shown that it is possible to predict, with sufficient accuracy, the mild wear on gear tooth surfaces using phenomenological models.
2. The wear on gear tooth surfaces has both a negative and a positive impact on the performance and service life of a gear transmission.
3. The mild wear is very likely to act as a catalyst for surface fatigue.
4. The wear behaviour of helical gears has been pointed out as well as some of its effects.

6. Further work

Further work could investigate the wear development of helical gears, as was done in paper D. However, no standardised test rig exists for this which means that anyone taking on this task would either have to build one or look at other machines incorporating helical gears. An FZG machine like that illustrated in figure 13 but incorporating helical gear wheels is a possibility if the wheels are fitted so as to neutralise the axial forces that come with helical gear wheels.

Further on, wear models and criteria for transition between wear mechanisms presents another area for research. This is important in order to be able to predict the wear development more accurately. Fatigue wear is also important, as is its connection to mild and moderate wear: the two mechanisms work in parallel from the start with the mild wear acting as a catalyst for fatigue wear.

Friction is another parameter needing further work for proper modelling. The influence of friction on the gear performance and gear surfaces also needs more investigation, which some have already started.
7 References


[34] Software KUGG ver. 2.24 from DAGO AB.