Storage of Bulk Solids in Silos
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Problems with the storage of bulk solids in bins and silos can be avoided if they are designed with respect to the flow properties of the bulk solid which has to be stored. The following essay considers the basic rules for the design of silos.

1. Stresses in silos

Figure 1 shows silos and the pressures and stresses, respectively, acting in the silos. While the pressure (for fluids we will use the word “pressure”) would increase linearly downwards if the silo would have been filled with a fluid (a), the course of the vertical stress (for bulk solids we will use the word “stress”) in a silo filled with a bulk solid is rather different (b,c): In the latter case in the vertical (cylindrical) section of the silo the vertical stress increases in a degressive way. If the height to diameter ratio of the silo is sufficiently large (usually: > 3), a constant vertical stress is attained. This means that the vertical stress will not increase further even if the filling height is much larger. The reason for this course are the shear stresses acting between the bulk solid and the silo walls even if the bulk solid is at rest. Due to the shear stresses, the silo walls carry a part of the weight of the bulk solid. A method for the calculation of the stress course in the vertical section was derived by Janssen already in 1895 [1]. This method is the basis for most present standards for the calculation of the load on silo walls for structural silo design [2-4].

The stresses acting in a hopper are different from those in the vertical section. Just after filling an empty silo, the so called filling stress state (also: active stress state, figure 1b) prevails, where the vertical stress in the hopper decreases less in the upper part of the hopper and then more near the imaginary hopper apex. As soon as some bulk solid is discharged for the first time after filling, the stresses in the hopper change and the so-called emptying stress state (also: passive stress state) prevails, figure 1c. When flowing downwards in the hopper, the bulk solid is compressed in the horizontal direction so that the walls of the hopper carry a larger part of the weight of the bulk solid and, hence, the vertical stress in the lower part of the hopper is clearly smaller than after filling. In the emptying stress state the vertical stresses in the lower part of the hopper are nearly proportional to the distance to the imaginary hopper tip or, in other words, the stresses are proportional to the local hopper diameter. This linear course of stress is called the radial stress field [7]. In principle, in the vertical section of the silo the stresses remain unchanged at discharge.

Two different modes of flow can be observed if a bulk solid is discharged from a silo: mass flow and funnel flow (figure 2a). In case of mass flow, the whole contents of the silo are in motion at discharge. Mass flow is only possible, if the hopper walls are sufficiently steep and/or smooth, and the bulk solid is discharged across the whole outlet opening. If a hopper wall is too flat or too rough, funnel flow will appear. In case of funnel flow (figure 2b), only that bulk solid is in motion first, which is placed in the area more or less above the outlet. The bulk solid adjacent to the hopper walls remains at rest and is called "dead" or "stagnant" zone. This bulk solid can be discharged only when the silo is emptied completely. The dead zones can reach the surface of the bulk solid filling so that funnel flow becomes obviously when observing the surface. It is possible as well that the dead zones are located only in the lower part of the silo so that funnel flow cannot be recognised by observing the surface of the silo filling.
3. Flow Problems

Typical problems which occur at the storage of bulk solids are:

**Arching**: If a stable arch is formed above the outlet so that the flow of the bulk solid is stopped, then this situation is called arching (figure 3a). In case of fine grained, cohesive bulk solid, the reason of arching is the strength (unconfined yield strength) of the bulk solid which is caused by the adhesion forces acting between the particles. In case of coarse grained bulk solid, arching is caused by blocking of single particles. Arching can be prevented by sufficiently large outlets.

**Ratholing** occurs in case of funnel flow if only the bulk solid above the outlet is flowing out, and the remaining bulk solid - the dead zones - keeps on its place and forms the rathole. The reason for this is the strength (unconfined yield strength) of the bulk solid. If the bulk solid consolidates increasingly with increasing period of storage at rest, the risk of ratholing increases. If a funnel flow silo is not emptied completely in sufficiently small regular time intervals, the period of storage at rest can become very large thus causing a strong time consolidation.
Irregular flow occurs if arches and ratholes are formed and collapse alternately. Thereby fine grained bulk solids can become fluidized when falling downwards to the outlet opening, so that they flow out of the silo like a fluid. This behaviour is called flooding. Flooding can cause a lot of dust, a continuous discharge becomes impossible.

Wide residence time distribution: If dead zones are formed (funnel flow), the bulk solid in this zones is discharged only at the complete emptying of the silo, whereas bulk solid, which is filled in later, but located closer to the axis of the silo, is discharged earlier. Because of that, a wide distribution of residence time appears which is disadvantageous in some cases (e.g. in case of storage of food or other products changing their properties with time).

Segregation: If a heap is formed on the bulk solids' surface at filling of the silo, segregation is possible according to particle size or particle density (figure 3c). In case of centric filling as shown in figure 3c, the larger particles accumulate close to the silo walls, while the smaller particles collect in the centre. In case of funnel flow, the finer particles, which are placed close to the centre, are discharged first while the coarser particles are discharged at the end. If such a silo is used, for example, as a buffer for a packing machine, this behaviour will yield to different particle size distributions in each packing. In case of a mass flow, the bulk solid will segregate at filling in the same manner, but it will become „remixed“ when flowing downwards in the hopper. Therewith, at mass flow the segregation effect described above is reduced significantly.
In a funnel flow silo, all problems mentioned above can occur generally, while in case of mass flow only arching has to be considered: segregation, ratholing, irregular flow and flooding of the bulk solid do not appear in a well designed mass flow silo. The residence time distribution of a mass flow silo is narrow, because it acts as a „first in - first out“ system (see figure 2a).

Two steps are necessary for the design of mass flow silos: The calculation of the required hopper slope which ensures mass flow, and the determination of the minimum outlet size to prevent arching.

4. Silo design

The flow behaviour of a bulk solid is defined by several well-defined parameters [2,5-8,21]. In general, these are the bulk density $\rho_b$, the effective angle of internal friction $\phi_e$ (a measure for the internal friction of the bulk solid at stationary flow), the unconfined yield strength $\sigma_c$, and the wall friction angle $\phi_x$. For mass flow design, the wall friction angle $\phi_x$ is the most important parameter, whereby the unconfined yield strength $\sigma_c$ is the most important parameter regarding arching. The wall friction angle $\phi_x$ is defined as the friction angle between the surface of the silo wall and the corresponding bulk solid. The unconfined yield strength $\sigma_c$ is the compressive strength of a bulk solid. It has to be taken into account that all these parameters are dependent on the stress level, represented by the consolidation stress $\sigma_1$ [2,5-8,21].

The parameters mentioned are measured in dependency on the consolidation stress with shear testers [2,5-8,21], e.g. with the Jenike shear tester or a ring shear tester. The hopper slope required for mass flow and the minimum outlet size to prevent arching can be calculated with the measured values using Jenikes' theory [7]. This method showed its validity in many cases in more than 35 years.

The borders between funnel and mass flow, which result from the calculations of Jenike [7], are shown in figure 4a for the wedge shaped hopper and in figure 4b for the conical hopper. In the diagrams the wall friction angle $\phi_x$ is drawn over the hopper slope angle $\Theta$ measured against the vertical. The effective angle of internal friction $\phi_e$, which is a measure of the internal friction of the bulk solid, is the parameter of the mass flow/funnel flow borderlines. The borderlines separate all pairs of values leading to mass flow from those leading to funnel flow.
Conditions which lie within the borderline yield mass flow whereas funnel flow is present in case of conditions outside of the borderline. If the wall friction angle $\phi_x$ and the effective angle of internal friction $\phi_e$ are known (measured with a shear tester, e.g. with the ring shear tester), the maximum slope angle $\Theta$ of the hopper wall against the vertical which ensures mass flow can be determined with this diagram. The courses of the borderlines indicate, that the larger the wall...
friction angle $\phi$ is, the steeper (smaller $\Theta$) the hopper has to be for mass flow. The wedge shaped hopper allows a somewhat (often $8^\circ$ to $10^\circ$) larger slope angle $\Theta$ against the vertical with the same material properties. That means that the walls of a wedge shaped mass flow hopper can be flatter than the walls of a conical mass flow hopper [7,12].

When bulk solid is discharged from a mass flow silo, the radial stress field prevails in the hopper as already described in section 1 (see figure 1c). In the hopper (at least beneath a sufficiently large distance from the vertical section) the major principal stress $\sigma_1$ is proportional to the local hopper diameter (figure 5). It decreases to zero towards the imaginary hopper apex. The stress $\sigma_1$ acts as a consolidation stress thus determining the properties of the bulk solid, e.g. the bulk density $\rho_b$ and the unconfined yield strength $\sigma_c$. The unconfined yield strength $\sigma_c$ of a bulk solid can be measured for each major principal stress (consolidation stress) $\sigma_1$ (see [21]). The function $\sigma_c = f(\sigma_1)$ (figure 6) is called the flow function. Usually, the unconfined yield strength increases with the consolidation stress. If the flow function has been measured, the unconfined yield strength $\sigma_c$ can be drawn in figure 5 at each position of the hopper.

$\sigma_1'$ is the bearing stress acting where an imaginary stable arch of bulk solid is carried by the hopper walls. $\sigma_1'$ is proportional to the local hopper diameter such as $\sigma_1$. An arch can only be stable in that area of the hopper where the unconfined yield strength $\sigma_c$ is larger than the bearing stress $\sigma_1'$. This is the case beneath the point of intersection of the $\sigma_c$ curve with the $\sigma_1'$ curve. Above that intersection the unconfined yield strength is smaller than the bearing stress of the arch. In this case, the unconfined yield strength is not large enough to support an arch, i.e. an
arch would not be stable at this position. The point of intersection indicates the lowest possible position in the hopper (height $h^*$, figure 5) for an outlet opening large enough to avoid arching. The diameter of this minimum outlet opening is called $d_{crit}$. If a smaller outlet opening would be chosen, $h^*$ indicates up to where flow promoting devices have to be installed beginning at the outlet.

![Flow function and time flow function](image)

Figure 6: Flow function and time flow function

Some bulk solids tend to consolidate increasingly with the period of storage at rest (time consolidation [8,21]). It can be found a time flow function $\sigma_{ct} = f(\sigma_1)$ (figure 6) for each storage time analogously to the flow function. If the time flow function would be drawn in figure 5 then this would yield to a point of intersection of the $\sigma_1'$-curve and the $\sigma_{ct}$-curve, which would be above the already determined point of intersection of the $\sigma_{1c}$- and $\sigma_1'$-curves. This means that larger outlets are required to prevent arching with increasing storage time at rest.

For practical silo design, equations or diagrams derived by Jenike [7] are used to determine the stresses $\sigma_1$ and $\sigma_1'$ in dependence on the flow properties measured ($\phi_e$, $\phi_x$, $\rho_b$) and the silo geometry ($\theta$). With this means the minimum outlet sizes of conical as well as wedge-shaped hoppers can be calculated. Furthermore, the minimum outlet sizes for avoiding ratholing at funnel flow can be determined [7].

5. Choice of the hopper geometry

Figure 7 [7,9,12] shows some opportunities to design mass flow silos. The calculations of Jenike (see design diagrams, figure 4) refer to conical hoppers (a) and wedge shaped hoppers (b). In case of these basic hopper forms, the maximum slope angles of the walls to achieve mass flow ($\Theta_{ax}$ in case of a conical, $\Theta_{eb}$ in case of a wedge-shaped hopper) and the outlet dimensions ($d$, $b$) to prevent arching can be determined. In case of the wedge shaped hopper it is assumed, that
the influence of the vertical end walls can be neglected if the length of the outlet $L$ is at least three times the width $b$.

The variants c and d are advantageous as well to ensure mass flow if the maximum slope angles as indicated in the figure are not exceeded. The pyramidal hopper geometry (e) is disadvantageous because the bulk solid has to flow from the top in the edges of the hopper and in the edges to the outlet. Thus, the bulk solid has to overcome wall friction at two sides what supports the formation of dead zones. If mass flow has to be achieved with such a hopper geometry, the edges have to be rounded on the inside, and the maximum slope angle against the vertical of the edges must not exceed $\Theta_{ax}$. Because the walls of a pyramidal hopper are always steeper than the adjacent edges, a pyramidal mass flow hopper is steeper than a conical hopper for a specific bulk solid. Variant f is just a transition from a cylindrical section to a square outlet. In this case, the slope of the hopper walls against the vertical must not exceed $\Theta_{ax}$ at any position.

In order to achieve mass flow, variants e and f must have the steepest walls. The conical hopper (a) can be designed more shallow, and the largest slope angles measured against the vertical can
be achieved with geometry b,c or d (wedge-shaped hoppers). In some industries non-symmetrical silos are preferred (e.g. pyramidal hoppers with differently sloped walls). From the view of mass flow design, there is no reason to build such silos. If mass flow has to be achieved, symmetrical hoppers usually require the lowest height for the transition from the silo cross-section to the outlet cross-section to achieve mass flow [10].

6. Application of the results on the design of silos

In section 4 the silo design procedure due to the theory of Jenike [7] was described in a shortened way. Further details and information can be given besides the determination of the hopper slope for mass flow and the size of the outlet to prevent arching. Some examples are listed shortly in the following (further examples of silo design: [17,18,22,23]):

Details about hopper slope and size of outlet for different hopper forms (see figure 7) and wall materials. Because of that, a comparison of manufacturing costs of different hopper forms and hopper materials is possible [18,19]. It can be find out, for example, whether lining of the hopper walls (e.g. with cold rolled stainless steel sheets) is useful regarding the costs.

If the mass flow design yields an extremely steep mass flow hopper, or if in the case of a retrofit of an existing silo mass flow should be achieved without modifying the (too shallow) hopper walls, specially suited installations can be dimensioned on the basis of the measured flow properties and Jenikes' theory [15,16].

In case of varying material properties (e.g. moisture [10,20]), it is possible to find out with shear tests which conditions would yield the worst flow properties. If the silo is designed for these conditions, proper operation is ensured in any case.

In case of bulk solids which tend to time consolidation, it can be stated quantitatively which size of outlet is necessary to avoid arching in dependence on the storage time at rest. A mass flow silo provides the opportunity to keep the bulk solid in motion by regular discharge (and recirculation, if possible) of a small amount of bulk solid. In this way, the time consolidation and, hence, the size of the outlet [18] can be limited.

With flow property tests (shear tests), the influence of additives (e.g. flow agents) can be determined in order to find the optimal mixture [13,18].

In case of the storage of bulk solids sensitive to attrition or stresses as present in a silo, the limit stress can be examined above which that danger exists [22,23]. Because of those results, the silo can be designed in that manner that no stresses will occur which would have a negative influence on the quality of the product.
To avoid vibrations emerging during discharge of a bulk solid, specially-suited installations can be designed (e.g. discharge tubes) [14,18].

In general, shear tests are also applied for quality control and flowability tests [13].

7 Summary
The design of silos in order to obtain reliable flow is possible on the basis of measured material properties and calculation methods. Because badly designed silos can yield operational problems and a decrease of the product quality, the geometry of silos should be determined always on the basis of the material properties. The expenses for testing and silo design are small compared to the costs of loss of production, quality problems and retrofits.